On Construction of Natural Numbers in Genetic Phenomenology (1)
—A Phenomenological Method for Grounding Naturalism—

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0. Introduction

In The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy (Die Krisis der europäischen Wissenschaften und die transzendentele Phänomenologie: hereunder abbreviated as Crisis), Edmund Husserl described mathematics and number-based natural sciences (such as physics) as a “garb of ideas” [ideenkleid] that cover the “life-world” [lebenswelt] (vgl. HuaVI, S. 51f). When discussing the relationship between life [leben] and ideas [idee], Husserl wrote that numbers, for example, exist in themselves [sein an sich] (HuaVI, S. 54) as ideas before being experienced (vgl. HuaVI, §9- i). However, Husserl believed that since numbers are refined through the human act of “measuring,” they have been constituted intersubjectively through history (vgl. HuaVI, S. 23ff). Thus, according to Husserl’s approach, mathematics is something that originated from our life-world.

So, if mathematics has its origin in the life-world, then it follows that it is something that is genetically acquire from our real life experiences. If this is the case, then it should be possible to explain phenomenologically the origin of mathematical consciousness within our minds. The issue here is different from the type of issue above, which would involve a discussion on intersubjective constitution relating to the history of mathematical concepts or the origin of mathematics as seen from a macroscopic viewpoint. However, inasmuch as the intersubjective constitution of numbers themselves and mathematical concepts is mutually tied to the constitution of such objects in the individual subject, then anyone inquiring into the origin of mathematics will be required to investigate them both. In other words, it is also necessary to discuss the meaning of ‘number’ in the individual subject and how it is constituted and cognized therein (for example, mathematical objects, numbers themselves, etc.) in terms of generative constitutive processes of consciousness (the constitutive operations [Leistung] of time-consciousness [retention, protention], association, awakening, pairing, etc.). I will therefore focus on the generative constitution of number that takes place in passive synthesis, and discuss the underlying essential regularity thereof.

In view of the above, I will proceed as follows: 1) With an understanding that, according to Husserl, the operations of time-consciousness function implicatively in the noesis (conscious act) of “counting,” I will focus on the noesis of counting and the generation of its corresponding noema (conscious content), and explain the same as being a constitutive process based on retention and protention. I will then turn my attention to the issue of the conscious constitution of the conceptual object of “number” and proceed as follows: 2) With the understanding that peano axioms formulate natural numbers as basic numbers, I will discuss mathematical induction, a method used to prove a given statement for all natural numbers (as formulated by the peano axioms), and argue that this can be related to the time-consciousness of futural horizon, by which such thinking is made possible. Through these two processes, I will
be able to analyze natural numbers through a generative phenomenology framework and give an account of the constitutive process thereof. Finally, this generative phenomenological analysis of mathematics will contribute to the argument that naturalism, which uses mathematics as a descriptive method, must be founded in phenomenology, or the primordial living-world.

1. A Phenomenological Discussion of the Relationship between “Counting” and “Number”

a) Constitution of Cardinal Numbers – Collective Combination

In *Philosophy of Arithmetic [Philosophie der Arithmetik]*, Husserl conducted a psychological analysis of the concept of cardinal numbers [*kardinalzahlen, Anzahl*], a basic concept in arithmetic; and he sought to ground the formation of this concept in psychical processes (vgl. HuaXII, S. 294f.). A distinction can be made between sets of numbers⁶ (cardinal numbers), which are used to describe the quantities of things, and ordinal numbers [*ordnungszahlen*], which are used to describe the sequence of things. Husserl believed that ordinal numbers are founded on cardinal numbers, writing that “series [reihen] are ordered sets” (HuaXII, S. 11).⁷ Generally speaking, the reason for supporting the view that cardinal numbers are the foundation is that the concept of set is treated as the justifiable object of mathematics.⁸ Therefore, at the time *Philosophy of Arithmetic* was published, Husserl believed that cardinal numbers rather than ordinal numbers are essential in order to grasp the concept of natural numbers.⁹

What did Husserl actually state in *Philosophy of Arithmetic*? If I am asked about the number of apples on a table, for example, I would have to answer how many [*wievel*] apples there are. I would, on this occasion, be *counting* each apple (1 apple, 2 apples, 3 apples, and so on). By counting in this way, I would *combine* to get the aggregate [*Inbegriff*] “number” (total number) of apples. This aggregate is the set or “collection” [*kollektion*] of the individual objects counted (vgl. HuaXII, Kap. I). Husserl used the term “collective combination” [*kollektive verbindung*] to denote the psychical act of combining to get the aggregate of such objects (HuaXII, S. 20).¹⁰ It is through this psychical act of collective combination that sets are formed. Furthermore, when abstracting the elements in such collections, representations of multiplicity [*vielheit*]—i.e. purely how many things are there—are established as “intentional objects” (HuaXII, S. 45), and when the determinate number is established by means of reflection, it becomes the general concept of cardinal number (vgl. HuaXII, S. 45).¹¹ Thus, Husserl locates the origin of number in a psychical operation (vgl. HuaXII, S. 71- 76).

However, one thing that requires further analysis is the constitutive process that takes place in the noesis of “counting.” As seen above, Husserl set out in *Philosophy of Arithmetic* the view that the concept of number is constituted by counting up the “quantity” of given objects by means of collective combination, abstraction, and then reflective objectification. However, what deserves attention here is the following statement that Husserl wrote when discussing the collective combination: “The temporal coexistence is indispensable for the representation of their plurality” (HuaXII, S. 24). What did Husserl mean by temporal coexistence?

b) Operations of Internal Time-consciousness that Function Implicitly in the Constitutive Acts of Numbers

In *Philosophy of Arithmetic*, Husserl developed the following discussion on the temporal coexistence of contents. First, with regard to how individual objects are finally counted up as aggregates and represented as higher-order objects, Husserl denied that when focusing on the final object, one starts again from the first object and then assembles [*zusammensetzen*] the represented contents as the aggregate (vgl. HuaXII, S. 24). Second, taking as an
example the representation of melody, which comprises individual tones arranged in a temporal sequence, Husserl argued that just as the representation of melody requires that the contents (tones) thereof be simultaneously “present-at-hand” [vorhandensein], the representation of totalities—i.e. multiplicities of objects—likewise demands that the contents thereof be represented simultaneously (ebd.) Third, Husserl had the following to say regarding the successive generation of given parts in a temporal sequence and the totalities of these assembled elements: “any collection presuppose an act of combining, each number counting...It is temporal succession and nothing else that characterize the multiplicity as multiplicity” (HuaXII, S. 25). In other words, according to what Husserl wrote, if we think in terms of the psychical act of counting and the constitution of the counted objects, then the acts of counting and collective combination forming the counted objects can indeed be considered a temporal constitution.

It is, of course, not the case that Husserl was conducting a phenomenological analysis of the constitution of time-consciousness when he wrote Philosophy of Arithmetic, but similar type of notes and arguments can be found in a text1212 that Husserl wrote on time-consciousness around this time (vgl. HuaX, Nr. 1). Furthermore, Husserl argued that when one counts up sequential contents and is conscious of this total as a higher-order representation, at the end point of this sequence there is no need to recount every datum and repeat the process starting from the first datum, and he also argued that at the end point one must retain the succession of contents up to that point in time. These two arguments resemble the crux of the criticism Husserl levied against Meinong during the course of lectures on time he delivered in Göttingen University in 1905 (vgl. HuaX, Nr. 29).13 Therefore, it can be argued that Husserl, while himself unaware that he had done so, had already set out in his notes about the constitution of consciousness the seedlings of an idea that would develop into his theory of a constitutive operation of time-consciousness, namely retention.

As we have seen above, Husserl believed that the counting up of given objects (collective combination), which forms the premise for the act of counting, involves temporal constitution. This temporal constitution is founded on nothing other than implicit [implizit] time-consciousness. For example, Husserl believed that such implicit constitution of inner time-consciousness develops as the foundation to active, higher order noesis, and in Experience and Judgment: Investigations in a Genealogy of Logic (Erfahrung und Urteil. Untersuchung zur Genealogie der Logik: hereunder abbreviated as EU.), a collection of the manuscripts Husserl composed during his last years, he wrote the following: “Every act of the ego, for example every act of simple apprehension of an object, appears in the temporal field as a temporally self-constituting datum” (EU. S. 122). In other words, the grounding of such acts in time-consciousness does not only apply to the relationship between the act of counting and collective combination. Lohmar, being interested in the issue of the origin of number, returned to the place where the act of counting originates. Citing the above statement from Experience and Judgment, he argued that “[the grounding of judgment in pre-predicative experiences] implies that judgment originates in the primordial ground of the givenness of experience, which is the givenness of individual objects and (therefore) all higher-order acts”15 (emphasis is mine). Lohmar then developed his argument as follows: “sets are pre-constituted in a collectivization [in einem Kollektionsakt], and then formed into objects through a regressive [im Rückgriff] grasp. Through the passage of collectivization, we know [wissen] that all the units are part of sets, and that all units are still-retaining-in-grasp [noch-im-griff].16 Lohmar thus identified the operation of “still-retaining-in-grasp”—which is certainly nothing other than the operation of retention—as the temporality in the act of counting.17

Richard Tieszen presented the example of counting a series of strokes (“ || || || || || || || ||”), and wrote the following: “At a certain stage we construct a unit, at a later stage we construct another unit, and then we look at this pair as an object.18 We could then at a still later stage construct another unit, taking the pair previously formed as a
term of a new pair, which gives another object. This process could be iterated as often as we like…In continuing the construction we are thus simply iterating the process (operation) of constructing a unit, i.e., adding a unit at each stage where we think of this as done at successive stages in time.” Tieszen represented this process in the following diagram:

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(( | ) | | )…
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The process Tieszen describes is nothing other than the constitution of time-consciousness. After the first vertical line is formed in the consciousness, the consciousness moves on to forming the second line while still retaining this first line. The use of nested parentheses in the above diagram helps us understand how the whole structure involved in determining number is “isomorphic across the cases” and perpetually constituted. In other words, we can understand it is constituted by retention (particularly the transverse intentionality of retention).

Tieszen has the following to say about such a constitution: “This type of retention is a very basic feature of consciousness which is involved in the constitution of any kind of object. It should be noted that it is not to be confused with an act of remembering. It is not actually an act that is performed but is rather a ‘passive’ feature associated with any act that has to do with our immediate awareness of an object.” Having read Tieszen’s words here, we should turn our attention to the fact that the abovementioned constitution of time is discussed as a process that takes place in passive synthesis. In short, we should relate Tieszen’s term “pair” to Husserl’s description of “pairing” [paarung] in passive synthesis. Pairing is what occurs when two units are constituted as a pair through association of similarity (similar object, dissimilar object) and the operation of time-consciousness in relation to continuation (present, just-having-been-present). Through being paired—i.e., by being retentively maintained—each datum undergoes a reciprocal awakening with the datum that follows to form, as it were, a “pre-collection.” In other words, before the act of counting is performed, the datums to be counted have been pre-constituted to provide the groundwork for the act. Therefore, it is passive synthesis, whereby a time series of phases is constituted by means of retention, which is what enables the active objectification of the units—i.e., the noesis of counting.

In this passive constitution, the operation of protention plays an important role. Protention is the impetus for the association of similarity during pairing, and it provides a basis for the temporal and noematic tendency (intentional direction). In other words, the operation of protention affects the ego to actively perform the “counting” of multiplicities that have been passively constituted. Having been thus affected, this act of counting entails the counting up of passively counted objects (pre-collection objects) one by one in a sequence constituted by time-consciousness.

c) The Relationship between Cardinal Numbers and Ordinal Numbers

Based on the above, we can now revisit the relationship between cardinal numbers and ordinal numbers. Tieszen writes that “the cardinal conception of number is founded on the ordinal conception … The basic idea is that to be conscious of a cardinal number \( n \) it is necessary to abstract from the ordering of units in a construction that would be constituted by virtue of successively running through the units.” Using these words as a springboard, let us clarify the various processes such as collective combination, constitution of time-consciousness, and constitution of consciousness of the number concept.

1) Passive synthesis of given hyletic data is generated. Given datums are pre-constituted as “noematic entities”
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(HuaXI, S. 75) by virtue of pairing, which takes place through the implicit operations of time-consciousness and reciprocal awakening in association. These pre-constituted entities provide the premise for the active operation of counting.

2) The pre-constituted “noematic entities” affect the ego. The ego then turns toward these entities and forms a mutual relationship with the noesis of “counting.” The act of consciousness is then performed in line with the build-up of successive phases as constituted by the implicit operation of time-consciousness (a datum, a subsequent datum, a further subsequent datum), particularly its transverse intentionality. The individual processes of the act of counting characterizes a given series as a series through the comprehensive unification of time-consciousness by horizontal intentionality.

3) At the point in time when protention becomes unfulfilled – i.e., at the final datum – representation of multiplicity as a higher-order collective combination is constituted alongside the series, which is formed through horizontal intentionality from the accumulation of datums in transverse intentionality (synthesis by pairing). Then, through abstraction and reflection, the number concept of simple numbers, or cardinal numbers, is generated.

Thus, based on the above processes, the concept of cardinal number—a concept that is fundamental to and prioritized in mathematics, particularly in set theory—can be considered as being formed from an exceptionally high-order constitution. The concept of ordinal number too is, no doubt, also a higher-order semantic content in its proper abstract sense of “nth place.” However, as can be garnered from the fact that Tieszen regarded in particular the series of counting-noesis described in step (2) as ordinal numbers, the constitution of an explicit time series of noesis can be considered as the constitution of time-consciousness. Having said that, when tracing further the constituting process of consciousness, if we suppose that the formation of an ordinal-valued time series prompts the establishment on the horizontal axis in Husserl’s time diagram, that is to say, the establishment of a series by means of the objectification of pre-constituted “pre-collection unities” in phases that are being formed on the vertical axis (as Yamaguchi argues), then it is at this level that the foundation of the sets of numbers (cardinal numbers) is found. The constitution at this level, which I have provisionally termed “pre-collection,” requires further discussion.

2. Mathematical Induction and the Horizon of Protention

a) Verifying Natural Numbers by the Peano Axioms

I have used the above phenomenological analysis to confirm the constitution of cardinal and ordinal numbers and their spontaneous order relationship. In this section, I will shift the focus to natural numbers, which are fundamental to mathematical thinking and mathematical objects. As I mentioned above, the objectification of pre-collective combinations at a passive synthesis level play a role in establishing the ordinal numerical concept. This being the case, natural numbers can be regarded in much the same way as ordinal numbers inasmuch as the numerical series continues as 1, 2, 3, and so on. However, how can the continuation of such a numerical series be determined? On what basis can we forecast that the series of numbers will continue? In discussing this issue, I will refer to the peano axioms, which mathematically verify the ordinal conception of natural numbers. Based on these axioms, I will present the problems concerning verification, and attempt to address these problems using a phenomenological analysis.

The peano axioms determine the set of all of natural numbers based on the following five postulates.
$N$ is a set of natural numbers $n$

1. 1 is a natural number. $1 \in N$

2. If any number $n$ is a natural number, then its successor $n'$ also is a natural number. $\forall \ n \in N \ [ \ n' \in N ]$

3. A successor of a natural number $n'$ cannot be 1. $\forall \ n \in N \ [ \ n' \neq 1 ]$

4. Two natural numbers $m$ and $n$ of which the successors are equal are themselves equal. $\forall \ m \in N \ \forall \ n \in N \ [ \ m' = n' \Rightarrow m = n ]$

5. Only that which meets the conditions of postulates 1 and 2 is a natural number (regarding $P(n)$ which contains a variable of a natural number $n$, if $P(1)$ is true, and if $P(k)$ is true for $P(1)$ and any natural number $k$, then when $P(k')$ holds true, $P(n)$ holds true for any natural number $n$.

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(P(1) \land \ \forall \ k \in N \ [ \ P(k) \Rightarrow P(k') ]) \Rightarrow \forall \ n \in N \ [ \ P(n) ]
\]

Going through in order, the first postulate determines that “1 belongs to set $N$.” The second postulate expresses the successor to $n$. If $n=1$, then $n'=1'$. Accordingly, the successor of 1 is 1'. The successor of 1 is neither 2 nor 1+1 because the number 2 and addition (+) have not yet been determined at this point (therefore, the successor of 1' is 1'''). The third postulate states that a successor number cannot be 1. The importance of this postulate lies in the fact that it determines alongside postulate 4 that 1 is the smallest natural number and that there are a succession of 's, one after the other, extending out from 1 at the beginning. With regard to postulate 4, $m' = n' \Rightarrow m = n$ determines the mathematical operation of ' for finding the successive numbers. This rule is necessary for eliminating the possibility that, for example, 1' and 1''' are the same. In other words, it determines that you proceed in series (in an ordinal way). According to these postulates, if, for example, we have, as part of a set, an element $a$ that cannot be traced from 1, then we cannot determine “natural numbers beginning from 1” with postulates 1 and 2 alone (there is the possibility that natural numbers could begin from an element not found within the axioms). Postulate 3 is therefore necessary. In addition, without postulate 4, there would be a possibility that an element $a$ overlaps a number somewhere – for example 1''' – meaning that there would be at least two numbers of the same value on the series of natural numbers. As for postulate 5, it is a theorem that verifies natural numbers inductively based on axioms 1 and 2. In other words, regarding $P(n)$ which contains a variable of a natural number $n$, if $P(1)$ is true as per axiom 1, and if $P(k')$ is true for an arbitrary natural number $k$ as per axiom 2, then the inference that $P(1')$ is also true holds true and is verified. This method of verification is mathematical induction. Through mathematical induction, it is possible to determine the whole set of natural numbers even without investigating each one of the natural numbers therein.

b) Phenomenological Analysis of Mathematical Induction

Based on the above, mathematical induction can be used to verify and comprehend an infinite series of natural numbers without having to count up directly. By using this method of logical inference, the totality of natural numbers, which do not yet exist in the consciousness, can be determined. The question that we must ask about this definition of natural numbers is undoubtedly how the noesis of such reasoning is constituted.

I already discussed the functioning of the operations of time-consciousness as part of my analysis of the constitution of consciousness of cardinal numbers. The act of counting holds true on the grounds that it is founded on sustaining retentions. It is indeed an actual experience, and through the act constitutes numbers, as it were, additively. However, the act of counting and the counted contents alone are insufficient to achieve the higher-order mathematical objectivity [representivity] of natural numbers. As demonstrated in the Peano axioms above, in order to form the concept of a whole set of natural numbers, it is necessary to have an axiom on “successive” numbers (postulate 2). In other words, the constitutive operation of consciousness concerning the actual situation of the “successive”—i.e.,
Protention—is exactly what concerns the phenomenological analysis. I will therefore discuss the reasoning that takes place in mathematical induction, and the actual situation of the axiom of successive numbers by which such reasoning is validated, in relation to the operation of protention.

Protention is an operation of time-consciousness. In Husserl’s words, “every process that constitutes its object originally is animated by protentions that emptily constitute what is coming as coming, that catch it and bring it toward fulfillment” (HuaX, S. 52). Protentions are closely linked with retentions. They are said to operation as their flipside or mirror image. The reason for this is that protention is non-salient (passive) intention that focuses on retained contents that are unfulfilled and emptied, and anticipates the fulfillment of this emptiness. The intentionality that awaits the fulfillment of this emptiness is indeed protention. Moreover, one important property of protention is “unfulfillment” (HuaXXXIII, S. 14). The contents whose fulfillment protention anticipates will not necessarily be fulfilled. However, even if they are not fulfilled, the protention will continue to anticipate their fulfillment. In short, through its unfulfillment, protention forms a horizon of a certain type of conscious tendency and empty intention; that is to say, a futural horizon. Thus, by virtue of the properties of protention, expectational consciousness of the future, which includes for example the modes of consciousness of negation, doubt, and possibility, are formed explicitly (vgl. HuaXI, Abschn. 1).

I will now attempt to discuss numerical succession and mathematical induction while giving consideration to the properties of protention shown above. When thinking of the “successive”, for example, in the constitutive process of time-consciousness for counting Tieszen’s series of strokes – “(( | ) | | | )…” – it can be argued that the protentional intention corresponds to the “…” part. In Tieszen’s discussion above, he only touched on the constitutive process of retention, but he did incorporate the protention process into his account of this time-consciousness constitution, even if unwittingly. This constitution does not only apply to objects such as Tieszen’s series of strokes. It is much the same in the Peano axioms, with 1 at the beginning, followed by another number, then another number, a further number, and so on (1, 1’, 1””, 1’’’…). The thing that is different is the involvement of the mathematical operation of ´. The performance of axiom 2 involves the focusing of protentional intention toward the phase that succeeds the retained content of 1, and the active mathematical operation of appending ´ to characterize this phase as a “successive” phase. Having been appended with ´, 1´ is further repeated in the flow of the constitution of time-consciousness, particularly in relation to protention, to form a series of succession through the performance of the same mathematical operations of retention and protention, the repetition of this operation is rendered possible, and a futural horizon unfurls to enable the ego to keep adding further successive numbers.

Husserl discussed protention in Analysis of Passive Synthesis, and he wrote the following: “If any kind of a, a sound for instance, is in the steady process of melting down new impressional phases, if it is, in other words, a course of continual connection corresponding to certain essential conditions in this original process of becoming, then a futural horizon, that is, an expectational horizon is immediately there along with it” (HuaXI, S. 186). The presence of this futural horizon prepares the way for a progressive process of operations, which are analogous to the previous process becoming and indicate expectation of the continual style of the course, and these operations are performed on the actual datums. It can therefore be understood that the “…” part in the series of strokes or successive numbers is the descriptive representation of the futural horizon.

It is certainly true to say that this constitutive process of consciousness allows the continual addition of successive numbers, but in our real-life experience, we could not count up all the elements in an infinite set of natural numbers.
The thing that can settle the difference between the anticipation that it is impossible in practice to count up all the numbers and the axioms of natural numbers is the regularity of the higher-order reasoning in mathematical induction. This regularity is, of course, defined and verified in a logical form, but the foundation of the performance of this logical form surely lies in our futural intention.

As we have seen from the above, the forecasting of continuation is established from a futural horizon, which is itself formed from protention or the unfulfilled intention thereof. Husserl wrote the following:

If we assume that the Circumstances $C$ would be constituted in a unitary manner in a previous situation of consciousness of the distant past and then a $q$ had ensued; and if we assume in addition to this that now in the currently present new situation of consciousness, the similar Circumstances $C'$ would have been (implicitly) repeated, then in the event that the previous $C$ and their $q$ have been awakened, the occurrence of $q'$ will now also be necessarily motivated as arriving. (…) I expect $q'$ here because I have experienced $q$ under similar circumstances, and this “because-thus” is given in evidence (HuaXI, S. 187f. emphasis is mine).

In short, being pre-constituted in passive synthesis, past experience and future expectation are both evidential and necessarily motivated. Regarding this “because-thus,” Husserl related the generation of the logical regularity which is inductive reasoning to the constitution of time-consciousness (protention) and passive synthesis (association and pairing), writing “correlatively [motivation correlated with evidence]: I infer ‘inductively’ in complete evidence the present, similar arrival from what has arrived under previous, similar circumstances. Like every inference, this too has necessity and yields in essential generalization an evident law of inference” (HuaXI, S. 188). We can therefore say that being founded on the experience of protention and futural horizon, the axioms concerning succession of natural numbers, the presence of a successive number, and the evidence of such from mathematical induction, ensue as “evidence” in higher-order noesis.

Based on what we have learned above, we can state that a series of natural numbers is constituted by a passive synthesis between the act of counting numbers and the implicit time-consciousness operation that underlines this act. However, it is not the case that I have discussed all the data concerning natural numbers. I did not discuss how consciousness of the privileged and predicative numbers 1 and 0 are constituted. I would like to further the phenomenological analysis of natural numbers, specifically by attempting to locate their primordial generation in the relationship between the fulfillment and unfulfillment of kinesthesia in infants, but because of space limitations, I cannot embark on such an attempt in this paper. I will leave it for another time.

3. Final Words

I will finally write some words about this paper’s ultimate aim. As I laid out at the beginning, in order to critique naturalism at a fundamental level, we must thoroughly examine the descriptive methods that the study of naturalism stand upon, namely mathematics and numbers themselves. This being the case, just as Husserl alerted readers to the possibility that nature itself is conceptualized by the Galilean mathematicalization of nature, it is necessary to found the leading concepts of mathematical natural science espoused by naturalism in the constitution of consciousness, the body, and the living-world, and to shed light on their origin. Doing so will lead to a fundamental change in the way highly abstract mathematical concepts are advanced and the attitudes of the people who use such concepts.

In reality, anyone who witnessed the tragedy of the nuclear disaster unfold would have become painfully aware of
the all too wide gulf between, on the one hand, natural science that made nuclear power possible in the first place and technology which also facilitated it, and, on the other hand, the raw reality of our consciousness. There is a need for substantial and fundamental reflection upon such pain within politics, law, and economics, but even more so within the greater philosophical contemplations. Unless this tragedy is comprehended with philosophical reflection and criticism, there will be no hope of finding a resolution and moving forward in the true sense. It is apt to point out that Husserl’s philosophical discussion of mathematical natural science which he undertook in Crisis came right on the eve of the Second World War. If we pause to consider how the relationship between people and science has provided us with blessings and misery in equal measure, we should recognize that it is undoubtedly the role of philosophy to radically question the relationship between the two. Understanding the role of philosophy to be such, I will aim to continue examining mathematics itself as something that links people and science together.

Literature

<Husserliana>


森村修「フッサールの「多様体の哲学」（1）—「多様体の哲学」の異端的系譜（4）—」異文化:論文編（13）、法
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Notes

1 Legend: For the quotations from *Husserliana* (Den Hagg, Kluwer Academic Publishers, 1950–), I have indicated in parentheses volume numbers with Roman numerals and page numbers with Arabic numerals. *Italic font* indicates text that is emphasized in the original text, and bold font indicates my emphasis. Supplementary annotations appear in square brackets. In addition, I have abbreviated *Experience and Judgment* [*Erfahrung und Urteil*] as *EU*.

2 There is general consensus on the historical flow of European mathematics—i.e. surveying began in Egypt, geometry was established with Euclid’s *Elements*, and then developed into Descartes’ analytical geometry. Particularly important in this historical flow is Descartes’ revolutionary achievement of integrating geometry and algebra (notwithstanding the fact that algebra came from Arabia). See Yoshida, Y., Seki, S., *Introduction to Mathematics*, Chikuma Gakugeibun, 2013.

3 In *Crisis*, Husserl discussed the historical development of mathematics (vgl. HuaVI, §9), and in his manuscript “Origin of Geometry” (an appendix in *Crisis*), he mentioned that mathematical concepts originate intersubjectively (vgl. HuaVI, S. 385).

4 In Appendix III of *Crisis*—“Origin of Geometry,” Husserl has the following to say about the intersubjective construction that is the historical development of not only mathematics, but all sciences: “every science is related to an open chain of the generations of those who work for and with one another, researchers either known or unknown to one another who are the productive subjectivity of the total living science” (HuaVI, S. 367).

5 For this reason, there is a need to examine the discussion of “the constitution of mathematical phenomena,” which was carried out in Husserl’s *Philosophy of Arithmetic* and “Origin of Geometry” (and also in Husserl’s *Experience and Judgment*, which was compiled by Ludwig Landgrebe). There are a number of works that exhaustively analyze these texts, and it is these works that should be referred to. They include Lohmar, D., *Phänomenologie der Mathematik*. Phaenomenologica 114, Kluwer Academic Publishers, Netherlands, 1989; Tieszen, R., *Mathematical Intuition, Phenomenology and Mathematical Knowledge*. Kluwer Academic Publishers, Netherlands, 1989, Nuki, S., “Mathematical Intuition: Husserlian Phenomenology and Philosophy of Mathematics,” *Husserl Studies*, No. 2, Grants-in-aid for Scientific Research 2002 (Basic Research B-1), Research Report on the Development of New Horizons in International Research on Husserlian Phenomenology based on New Data/ New Research, No. 2, 2004, 129–139, Suzuki, S., *Phenomenology of Mathematics*, Hosei University Press, 2013. Of these, a particularly important work in relation to Husserl’s theory of number is Nuki’s paper, in which he developed a discussion on the generation of number from the perspective of passive synthesis.

6 For example, in the case of the following two sets—{1, 2, 3} and {4, 5, 6}—while the elements themselves are not the same, the number of items are (mapping relationship. Making an arbitrary set of elements correspond with another set of elements). The number of elements in a set of elements is the cardinality, or the cardinal number (see Takeuchi, G., *What are Sets?* Kodansha, 2001, Chapter 2).

7 See Suzuki (2013) pp. 75–78 and footnote 76 on page 237. It is also worth seeing Morimura, O., “Husserl’s ‘Philosophy of Multiplicity’ (1) – The Heretical Genealogy of the ‘Philosophy of Multiplicity’ (4) –,” *Collection of Essays on Other Culture* (13), Bulletin of the Faculty of International Culture Studies, Hosei University, 2012, 183–220 and 198–199. Husserl himself wrote that “it is totally inadmissible to begin the systematic treatment of arithmetic with the series of natural numbers” (HusaXII, S. 399.)

8 With regard to the continuity of numbers, it is intuitively unproblematic to plot natural numbers, integers, or rational numbers along a number line, but there is an issue with irrational numbers (square root of 2, pi, etc.). Therefore, the method followed when expressing irrational numbers on a number line is to consider the limit of rational numbers. If the convergence of a numerical sequence (Cauchy sequence) is not a rational number, it will be an irrational number, but by being treated as the limit point, the irrational number is deemed intuitively continuous—i.e. positioned on a number line. Next, through
considering rational numbers and irrational numbers from a set theory approach, they will both be defined as real numbers that possess them as subsets. If numbers are dealt with as the cardinality of sets when they are viewed in terms of real numbers, they cannot increase infinitely as in potential infinity; instead, it is necessary to determine a beginning and end—as in actual infinity (when considering the continuity of a sequence of real numbers, the limit of irrational numbers [0.99999…, 3.141592…] will exist within the sequence). It is not known whether such irrational numbers do actually continue infinitely, and it is not actually possible to verify this. Therefore, it is possible to consider each irrational number as an element among the total set of irrational numbers. An understanding can then be gained by performing an abstraction from a comprehensive perspective, which is different from the level of intuitive confirmation, and formulating a viewpoint that differs from intuition (set theory). Thus, in mathematics, as far as real numbers are considered as cardinality (number of elements), it is possible to support set theory and, in turn, cardinal numbers, from a completion (ring theory) perspective.

Suzuki writes that Husserl’s outlook here was influenced by his mathematics teacher Karl Weierstrass’ work “Platonism in Mathematics.” Suzuki (2013), 1.3, 26–31.


See Nuki (2004), 135; and Morimura (2012), 202. On the concept of multiplicity, Nuki draws attention to the following: “Large numbers that deviate from ‘actual’ [eigentlich] number concepts can only be inactual [uneigentlich] or signative-symbolic.” Furthermore, Morimura points out that when he wrote Philosophy of Arithmetic, Husserl did not clearly distinguish representations from concepts. I will not go into detail here; I will instead leave it for a later paper.

According to R. Boehme, who compiled the tenth volume of Husserl’s complete works, this text was written in 1893 (vgl. HuaX, S. 137, Anm. 1).

The focus of Husserl’s critique of Meinong during his course of lectures concerned Meinong’s preconception that higher-order objects are limited to the point of the final phase of a temporal sequence, and the idea of the non-temporality of the experience of counting up the contents before reaching the final phase (vgl. HuaX, S. 219, 225). Husserl criticized these points on the basis that the account of phenomenological experience clearly reveals them to be false and irrational. Of course, this criticism of Meinong was made in the apprehension schema developed in Husserl’s Logical Investigations; therefore, concerning continuous awakenings and the constitution of the representations thereof, no consideration is given to retention, which establishes the continuity of sequentially occurring phases while on the other hand constituting a total temporal unity. However, Husserl attempted to solve this problem by setting forth the idea of “fresh memory” (“just-having-been-present”) (HuaX, S. 165) and “dual retention” (vgl. HuaX, Nr. 30–33).

For Husserl, implicit [implizit] was an important term for denoting the character of non-salient retention, and, as such, it indicates the non-saliency of internal time-consciousness and the accumulation of retained contents. In relation to this point, please refer to page 158 of my own work “Protention and Affection in ‘The Bernau Manuscripts’—Questioning the Necessity of Protention in the Constitution of the Stream of Consciousness,” Annual report of Phenomenology, No. 29, the Phenomenological Association of Japan (ed.) 2013, 157–166.


Nuki writes the following: “One focuses on each of the details or characteristics of the objects of perception, and one ‘still-retains-in-grasp’ [noch-im-griff] the nature of the object as revealed in such a process, thereby gaining an overall view. In the case of numbers, this operation is synonymous with ‘counting’ [Zählen],” Nuki, 2004, 135. See also vgl. Lohmar (1989).

Cf. Tieszen (1989), Chap. 5, or 6. For example, Tieszen writes that “what is relevant to the awareness of number is not anything about the particular objects numbered, but rather the structure of the cognitive processes involved in making determinations of number…”(Tieszen, 1989, 97) and “The idea of iteration or succession in time is of course an essential feature of the process of intuition according to the phenomenological description of this process” (Tieszen, 1989, 100).


Ibid. With regard to this process, Tieszen wrote the following: “At a certain time we construct a unit, and at a later time we construct another unit, so that we have a ‘first’ unit, and a ‘second’ unit. Then we look at or regard this pair as an object, as two units. After constructing a ‘third’ unit we take the pair previously formed as a term of a new pair, and then look at the resulting pair as an object, as three units” cf. Tieszen (1989), p. 106.


Transverse intentionality is one aspect of retention. The transverse intentionality of retention continually turns to equal and immanent units within the flux of temporal modifications to form continuity and change. Another aspect of retention is horizontal intentionality. The horizontal intentionality of retention concerns itself with the unity of this time flux. In other
Pairing means the constitution of binary relationships formed out of (primal) associations and operations of time-consciousness (retention and protention), and it is the essential regularity in passive synthesis. In other words, the similarity and continuity between two datums work together in synthesis, so that the two datums relate to each other and are formed as “one pair.” The generation of pairing is a primal form of the passive synthesis (vgl. HuaXIV, Nr. 35). With regard to this point, Nuki clearly laid out the following: “Pairing” is “passive synthesis” which operates spontaneously with absolutely no regard to the volition of the subject of cognition or action or to its motility. Time-consciousness is the structuring of our experiences themselves, and it operates even more passively than this passive synthesis” (Nuki, 2004, 136).

Reciprocal awakening is a passive synthesis that indicates the mutual arousal of the present and the past; that is to say the mutual arousal of newly given datums and empty representations sunken in the past (frames of meaning; these empty representations have reduced intuitiveness, and so the intention is unfulfilled. They are sensed as empty forms). To find out more about reciprocal awakening, see Yamaguchi, I., From Existence to Genesis, Chisen Shokan, 2005, 79-83.

In Analysis of Passive Synthesis, Husserl analyzes “associative awakenings” (HuaXI, S. 77). During this analysis, he turns his attention to the intentional directedness of associations (vgl. HuaXI, S. 76f). This directedness involves “tendency” and “[intention toward] fulfillment” (vgl. HuaXI, S. 83), which is surely the very essence of protention.

This could also be called pre-noema, that is to say, semantic content pre-constituted in passive synthesis. It does not become explicit semantic content until noesis is generated, and so it should be distinguished from full noema by being referred to as “[noema]-tic entity” or by placing the word noema in parenthesis.

Tieszen writes that “in general, at the nth stage in the construction where we reach the nth unit we shall, in looking at the new pair formed at that stage as an object, have n units” cf. Tieszen (1989), p. 106. In relation to this point, Nuki writes the following: “The one and only thing that is formed from the construction of time-consciousness in the iterative sensory procedures is ordinal number—i.e., what the ordinal position of the number presently being counted is. After the act of counting has been conducted once, the ego objectifies the counting operation once again, this time through remembering. As a result, the relationship of the cardinal number, which provides a one-dimensional and simultaneous overview of each counting stage, becomes clear and the quantity also becomes clear through the objectification of the overall counting process” Nuki (2004), 136-137.

For more on this point, refer to the account in Yamaguchi, I., Memory of Sense, Chizen Shoten, 214-215, and to diagrams 4 and 5.

In set theory, there are two types of ordinals. The first type of ordinals are deemed to have the function of arranging the elements of a finite set in order of natural numbers, and on this basis they are viewed as no different from natural numbers. The second type of ordinals are transfinite ordinals, which represent the order in infinite sets, particularly countable infinite sets (a set in which one can count off the elements and be able to conceive of the subsequent element), Takeuchi (2001), 76-93.

In [the Japanese translation of] On the Concept of Numbers [Sul Concetto di Numero], the axioms are represented as follows:

1. 《イチはある数である》
2. 《ある数の後に置かれた記号+はある数を与える》
3. 《もし a と b が二つの数で、それらの次のものが等しいならば、それらもまた等しい》
4. 《イチはどんな数の後にくることはない》
5. 《もし s がイチを含む類で、また s に属するものの次のものからなる類が s に含まれるならば、すべての数は類 s に含まれる》

1) 1 is a number; 2) A number with a + sign after it is also a number; 3) If the successor of a and b is the same, then a and b are the same; 4) 1 is not the successor of any number; 5) If s is a property of 1, and if the property of the successor of s is also s, then all numbers belong to property s. Peano, G., On the Concept of Numbers [Sul Concetto di Numero], Genealogy of Modern Mathematics 2, translated by Ono K., Umezawa, T., Kyoritsu Publishing, 1969, 101.) Axioms three and four are the other way round to the order I have used in the main text, but this poses no problem to the verification. Also, in this rendition, axiom 2 features the plus sign (+), whereas I have used the prime symbol (′).

Vgl. HuaXI, 186f.

With regard to “motivation” in passive synthesis, I have followed closely what is written in the Bernau Manuscripts about protention tendency and Affection (see Muto, 2013, 162-163).

When he wrote Philosophy of Arithmetic, Husserl believed that the numbers 0 and 1 did not originally belong in the natural number series and that they were added in later as secondary numbers. Based on such a view, he argued that the natural...
number series is not uniform (vgl. HuaXII, S. 129ff.) The idea that these numbers were added later is worth considering. It is certainly conceivable that their place of genesis may not be the same as the succession of numbers.
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